Modeling Elasticity: A Brief Survey of Price Elasticity of Demand Estimation Methods*

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Abstract- *For marketers and sales professionals, estimating price elasticities of their products is crucial for understanding sales and setting pricing strategies. Yet, given the variety of possible econometric models, the central question that arises as which one of them would be the most appropriate for elasticity measurement. This paper conducts a comprehensive empirical study of 104 weeks of sales (January 2016 to December 2017) for 340 Hair Care products sold in 11 retailers. Our first findings show that considering breakpoints and outliers ahead of using any econometric model significantly improves the output from the classical and most widely used models such as Ordinary Least Squares (OLS) and Quantile Regressions (QR). Moreover, we present two other innovative models, Quantile on Quantile Regression (QQR) and Gravity Center Regression (GCR) which could further eliminate the measurement bias given limited or even aggregated data and, assist with the marketing decision making processes.*

Keywords: *Price elasticity of demand; Breakpoint and Outlier; Quantile on Quantile Regression (QQR); Gravity Center Regression (GCR)*

JEL Classification: *C10; D40; L11; M31*

1. INTRODUCTION

In Economics, the law of demand tells us that there is a negative relationship between prices and quantities sold, i.e., the demand function is downward sloping. Moreover, there are two competing affects that influence the sellers' decision to increase (decrease) prices. When prices increase, the sellers' revenue increase due to the fact that each unit sold has a higher price (price effect). However, after a price increase, consumers could decide to purchase less, which will drive the revenue down (quantity effect). These two effects work against each other causing total revenue volatility and uncertainty. To determine which effect outweighs the other, people look at measures such as price elasticities that measure the responsiveness of unit sales to the changes of their corresponding prices. Recall that price elasticities simply measure the percentage change in unit sales given a small percentage change in prices.

Price elasticities play a central role in marketers and sale professionals' decision-making processes. They use these elasticities to determine their marketing campaigns and sales strategies, among other very important decisions.

In general, the demand for a good can either be elastic, inelastic or unit elastic. An elastic product is one which elasticity is smaller than -11 or greater than 1 in absolute values. In this case, a 1% increase in price will result in a more than 1 percent decrease in quantity sold; i.e., the quantity effect is stronger than the price effect. Under this circumstance, increasing price drag total revenue down. In another hand, inelastic products have a lower responsiveness to increases on their prices with price elasticities larger than -1 (or smaller than 1 in absolute value). E.g. a 1% increase in price results in a less than 1% decrease in quantity sold. In this last case, the price effect outweighs the quantity effect and a price increase could push the total revenue to go higher. Elasticity of Everyday Retail Price (EDRP) is used to set price strategies that help corporations increase sales, market share or profits, and ideally, all three.

In real life however, with aggregated data across multiple dataclasses (defined as unique retailer/product combinations), it is inevitable to estimate abnormally large and even positive elasticities (Blattberg and George, 1991)[1], which violate the law of demand.2 These results can be explained in different ways. For example, a single outlier could drive the real effect far away from its unbiased value and completely confuse the analysis that comes after; another data characteristic that

* We would like to thank Prof. Dominick Salvatore, Hrishikesh Vinod from Fordham University, David Butcher, Michael Surmeian from TABS analytics and several other colleagues for their helpful comments on earlier versions of this manuscript. The usual disclaimers apply.

[1] Recall that as soon as the demand function has a negative slope, the expected (theoretical) price elasticity should be negative.

[2] One other practical matter maybe worth mentioning is that for CPG/FMCG products often there are no more than 120 weeks available, which makes time series vulnerable to outliers if there are only a few shifts in EDRP.

contributes with erroneous estimates is the presence of structural changes (for example permanent changes in the EDRPs3) in the sample period. In these cases, and before applying any econometric model to estimate the elasticities, one should use a model to capture the breaks (effectively creating one -or more- subsamples).
Additionally, elasticities vary within different price ranges. In this sense, elasticities should be stronger when prices are higher (respect to competition or other metrics like percentage of clients' income, among others) since customers should be more sensitive for price shifts in a high-price region as opposed to price changes in a low-price region. Thus, using one single elasticity as a hint for EDRPs could provide misleading information used in marketing strategy decisions.
In an attempt to solve for the above-mentioned problems and improve measurement accuracy, in this paper we use four different approaches to estimate price elasticities: Ordinary Least Squares (OLS), Quantile Regression (QR), Quantile on Quantile Regression (QQR) and Gravity Center Regression (GCR). As we show, by simply applying a breakpoint and outlier detection model ahead improves the output from all econometrics' models when doing elasticity analysis. In addition, and in order to capture the potentially different dynamics of elasticities depending on high or low relative prices, we use Quantile Regression (QR) model to find elasticity estimates that correspond to different quantity levels and then, Quantile on Quantile Regression (QQR) model to capture the varying dependence structure that the different quantiles of price changes have on the different quantiles of quantity. Finally, we use a Gravity Center Regression (GCR) model, based on partial moment theory, to partition the joint distribution and create clusters that are hierarchical and partitional. By construction, GCR is the only model that always follows the law of demand theory, since it only uses data that falls in the II and IV quadrants of the partial moments of the data at hand.
Studies related to this paper are few. The most relevant work comes from Blattberg & George (1991)[1] and Montgomery (1997)[5]. Aiming to obtaining a robust price elasticities with respect to OLS model, these scholars apply Gibbs' sampling approach to estimate the parameters in a Hierarchical Bayesian Regression model.
Blattberg & George (1991)[1] used data on four bathroom tissue brands from three store chains and applied a shrinkage procedure based on empirical Bayes and hierarchical Bayes to shrink the chain-brand level OLS estimates toward a grand mean to avoid nonsensical estimates (positive elasticities). The limitation of this approach is that one needs three constraints before applying this model: the expected value of elasticity (regression coefficient) should be equal overall, equal across brands and equal across chains.

In a similar study Montgomery (1997)[5] focused on micro-marketing strategies by estimating store-level demand elasticity. He used a larger dataset containing 11 brands of refrigerated orange juice from 83 stores. Instead of assuming homogeneous stores, in his paper, the heterogeneous store level parameters were considered as a combination of chain level and store specific effects. Montgomery (1997)[5] include a new parameter ("demographic predictor") to link to the store specific heterogeneous characteristics to estimate cross-store estimates that are then shrunk toward a regression line4.
Even though all these researches show that elasticities estimated based on Gibbs sampling approach in a hierarchical Bayesian framework can yield better results and provide more stable measurement than conventional ordinary least squares (OLS) approach, their approach takes the form of a single conditional mean equation based on resampling result and as such fails to catch the dynamic changes of elasticity measures in each the overall price range. It is precisely the problem of static elasticity (or unique elasticity coefficient) that led us towards the use of novel models, such as QR and QQR approach that are used in this paper to trace elasticity dynamics under different price specifications. An extensive search of the quantitative marketing and econometric literature leads us to believe that our research provides the first complete set of elasticity model testing using the aforementioned econometric models.
Our findings are useful for marketing decisions by suggesting elasticities that are not only better estimated by capturing a more complete dependence structure between prices and quantities. Although other factors, such as substitutes (alternative choices), consumer income effect (proportion of a family's income) and different time horizon (long-term versus short-term effect), also contribute to elasticity variation, the primary aim of this paper is to focus on price elasticity measurement and model comparison with different type of econometric analysis. Even though the techniques presented here can potentially be applied to other scenarios as will become clear throughout the paper.
The rest of the paper is organized as follows. In the next section we provide a brief overview of elasticity definition and its relationship with total revenues. In section 3, we introduce four econometrics models that we use in this paper, Ordinary Least Squares (OLS), Quantile

[3] EDRP (Everyday Retail Prices) are those prices that do not have any promotional activities incorporated, i.e. is the price in the absence of any promotions.

[4] Indeed, Montgomery (1997) is very similar to Blattberg and George (1991) paper. However, Blattberg and George (1991) assume that the expected value of elasticity (regression coefficient) should be equal overall, equal across brands and equal across chains. Montgomery paper highlights firm heterogeneous property by using a new demographic variable to show store differences using several variables like latitude, near highway, and so on. Then they propose a marketing strategy based on each specific store.

Regression (QR), Quantile on Quantile Regression (QQR) (GCR) and, explain the outlier and structural break effects on elasticities. The results of our modeling efforts are then presented and discussed followed with conclusions pertaining to future work.

2. PRICE ELASTICITY

In this section we briefly describe price elasticity, discuss its relationship with total revenues and demonstrate how this measure informs better marketing decisions.

2.1 What is Price Elasticity?

Price elasticity[5] measures the changes in demand for a product in reaction to the changes of that product's price (keeping constant all the other variables that affect the demand function). Mathematically:

$$\varepsilon = \frac{\frac{dQ}{Q}}{\frac{dP}{P}} = \frac{d\ln Q}{d\ln P} \# (1)$$

Where ε is the price elasticity of demand, Q is the quantity demanded and P represents the selling price.

After collecting the prices and quantities (P, Q), we can obtain the demand elasticity through a regression function[6]:

$$\ln Q = a + b\ln P + e \# (2)$$

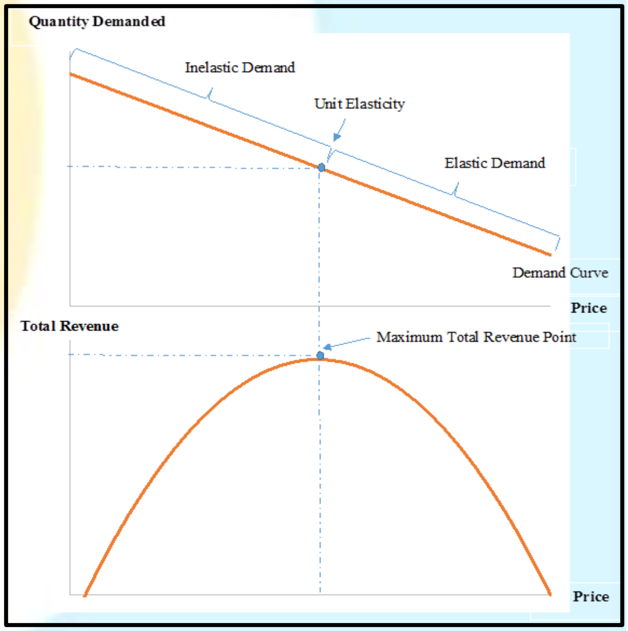
Thus, the slope term (b) is an estimate of the price elasticity (ε) of the demand curve.[7] The errors e are assumed to be i.i.d. From equation (1) above, it is clear that ε should be negative given the law of demand and, both analytical and empirical results confirm this. When the absolute value of this ratio is greater than one, the product is elastic, and demand declines more as price increases. In another hand, with an absolute value of ε less than one, the demand for a product does change but proportionally less than the percentage change in price.

2.2 The Relationship Between Elasticity Demand and Total Revenue

The mathematical link between total revenue and elasticity comes from the price elasticity of demand and Gravity Center Regression formula presented in Equation (1). Since the total revenue is given by

$$TR = P \times Q = P \times f(P) \# (3)$$

Figure 1: Relationship between elasticity demand and total revenue. Note: with unit elastic corresponding to the middle of the demand curve, everything to the left is inelastic and everything to the right is elastic. Revenue is maximized at the point where elasticity is unit elastic.



$$\frac{dTR}{dP} = Q + f'(P) \times P = Q\left(1 + f'(P) \times \frac{P}{Q}\right) = Q\left(1 + \frac{dQ}{dP} \times \frac{P}{Q}\right) = Q(1+\varepsilon) \# (4)$$

Where quantity demanded Q is a function of price P.

Therefore, Where ε represents the price elasticity. If demand is elastic $(\varepsilon < -1)$ then $\frac{dTR}{dP} < 0$. In this case, price and total revenue move in opposite directions.

This means that when we decrease prices, the total revenues increase. If instead, demand is inelastic $(\varepsilon > -1)$ then $\frac{dTR}{dP} > 0$: price and total revenue change in the same direction. Higher revenue could be obtained by pushing up the prices. If demand is unit elastic $(\varepsilon = -1)$, then an increase in price has no influence on the total revenue.

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[5] In this paper we are focusing our efforts is EDRP elasticities.

[6] Indeed, elasticity is a static concept measured around a current EDRP. In equilibrium (considering prices of all complement and substitute products and other economic/behavioral characteristics of buyers as well as other supply considerations, all comparable products should have the same price with unit elasticity (elasticity of -1). I.e. all producers or sellers maximize their revenues and at this EDRP point (the steady state) there is no incentive to permanently move the prices.

[7] To see this, taking the derivative of equation (2): $\frac{d(\ln Q)}{d(\ln P)} = \frac{\frac{dQ}{Q}}{\frac{dP}{P}} = b$. Note that the formula for b is the same as the one for the elasticity (ε). Thus, the coefficient b represents the price elasticity.

3. DATA AND METHODOLOGY

We employ 104 weeks of sales for 340 products from 11 retailers.[8] The sample period extends from January 2016 to December 2017 with total number of 219,024 observations for each variable (prices and units sold). We apply four econometric models to a total of 2106 (219,024/104) Dataclasses.

Before running our regression, we first clean our dataset by eliminating NAN, INF or missing price or quantity records. This reduces the sample size for each good but provides more robust results. We compute log price (lnP) and quantity (lnQ) to achieve elasticity directly. With limited and aggregated data, it is sometimes inevitable to obtain biased elasticity result due to several small sample issues that we introduce in next sections. We then discuss how to eliminate these biased results with the use of different models and cleaning or adjustment procedures.

3.1 Ordinary Least Squares (OLS) with Outliers and/or Breakpoints

The ordinary least squares model is the easiest and most used model in applied demand elasticity analysis. We take lnP as our independent variable and lnQ as dependent variable, ln stands for the natural logarithm, and run linear regression with:

$$lnQ = a + blnP + e \#(5)$$

Based on the sign of the coefficient b, we then test for the presence of outliers and/or breakpoints that could be present in the data. For example, abnormal positive price elasticity ($b > 0$) could be caused by outliers or structural breaks. Of course, one can perform this analysis before running the regression. However, with limited analysis information and the large number of products performing the analysis one by one would imply the use of significant human capital and technical resources. In this paper, we test for all scenarios with 1) breakpoint and outlier; 2) breakpoint only; 3) outlier only; 4) no breakpoint or outlier.

An example of a product with a structural break is shown in Figure 2[9]. The reason we get a strict positive slope ($b = 1.0021$) in this case is because there is a structural change in our dataset found on the quantity sold.

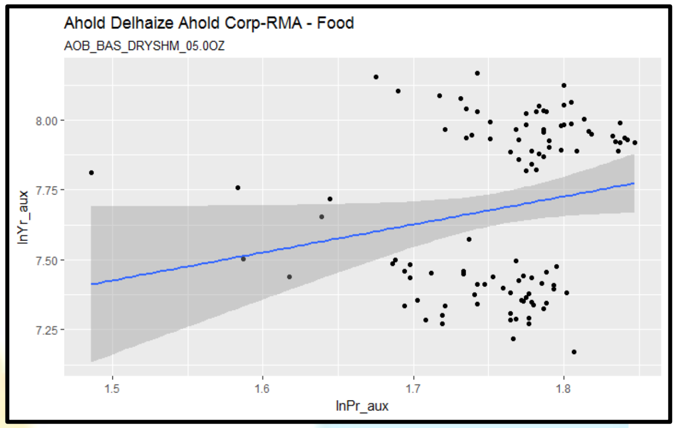

Figure 2: OLS result ignores structure break effect, estimated elasticity equals to 1.0021. Note: the black dots are $(\ln P, \ln Q)$ *pairs.* $\ln P$ *is shown on the x-axis and* $\ln Q$ *is shown on the y-axis; regression line showed in blue and, the grey shadow represents the range under 95% confidence interval.*

Based on the positive sign of the elasticity coefficient (β), we then apply the R package "breakpoint package" to detect and obtain the position(s) of the break point(s).

The breakpoint method implements variants of the Cross-Entropy (CE) method proposed in Priyadarshana and Sofronov (2012, 2015)[6][7] which is a model-based stochastic optimization procedure to obtain the estimates on both the number and the corresponding locations of the breakpoints in biological sequences of continuous and discrete measurements.

In our dataset we have found a maximum of only one break point (in the first week in 2017) based on lnQ. However, the procedure is able to capture more break points. Once this is considered we apply OLS regression equation with one dummy variable $d1$ that equals 1 for observations before break point (included) and 0 otherwise. Thus, our regression function becomes:

$$lnQ = a + \beta_1 lnP + \beta_2 d1 + \beta_3 d1 lnP + e \qquad (6)$$

Where $d1$ is the dummy variable described before. Note that Equation (6) is able to track not only changes in the y-intercept but also changes in the slope (elasticity) coefficients.

One problem with our dataset is that after adding the dummy variable, for some certain products, we enter into multi-collinearity problem in a linear regression function. When this happens, we try regression function $lnQ = a + \beta_1 lnP + \beta_2 d1 + e$[10] instead or, use equation (5) if multi-collinearity problem still exists. In all cases we analyze the final elasticity estimates and look for its economic soundness. In the example presented in Figure 2, the structure break happened on the 53th observation[11] (1/8/2017), from where sales increased a lot thereafter. We apply OLS again including dummy variable this time

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[8] Ahold, CVS, Kmart, Kroger, Meijer, Publix, Rite Aid, Southeastern Grocers, Wakefern, Walgreens and Walmart.

[9] For product AOB_BAS_DRYSHM_05.0OZ from retailer Ahold Corp

[10] For certain products, we arrive multi-collinearity problem due to limited dataset size. Instead of trying regression function $\ln Q = a + \beta_1 \ln P + \beta_2 d1 \ln P + e$ we use $\ln Q = a + \beta_1 \ln P + \beta_2 d1 + e$.

[11] Data available upon request.

to distinguish the different effect. The fitted lines are presented in Figure 3.

We get elasticity equals to -1.348 before the breaking point (blue line) and -0.8682 after the breaking point (red line). Both elasticities conform to the law of demand and improve a lot from previous result (slope $b = 1.0021$) estimated using the one-period OLS model. The difference between Figures 2 and 3 highlight a significant challenge for this aggregated data, namely Simpson's paradox presented in Simpson (1951) whereby trends are present in individual groups of data (product data for specific retailer locations), yet reverse when combined (overall product data for each of the 11 retailers).

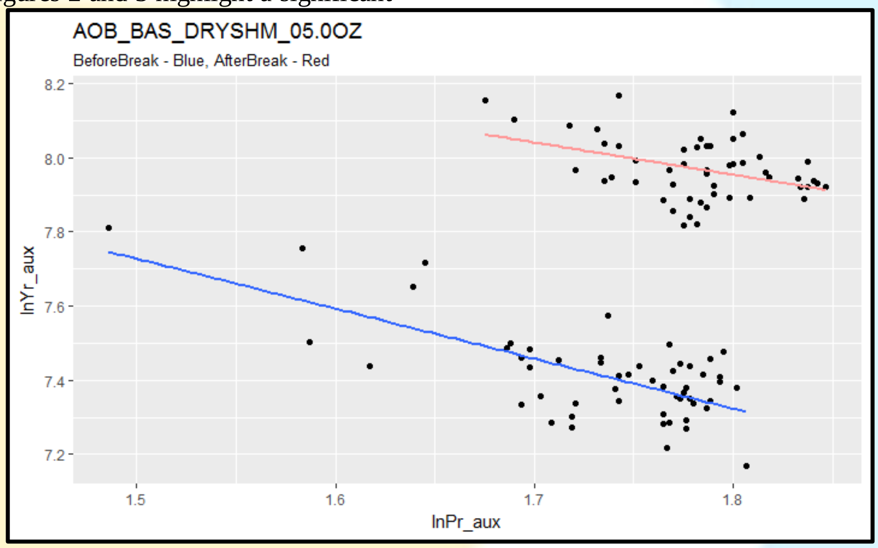


Figure 3: OLS result that considers one structural break. Separate dataset into two subsets from 1/8/2017, regression fit for data happened before 1/8/2017 show in blue and red if after. Estimated elasticities equal to -1.348 (before) and -0.8682 (after).

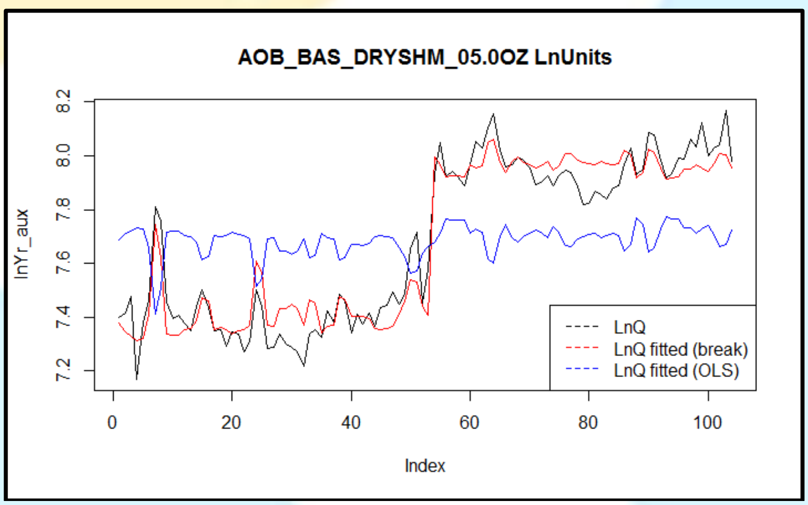


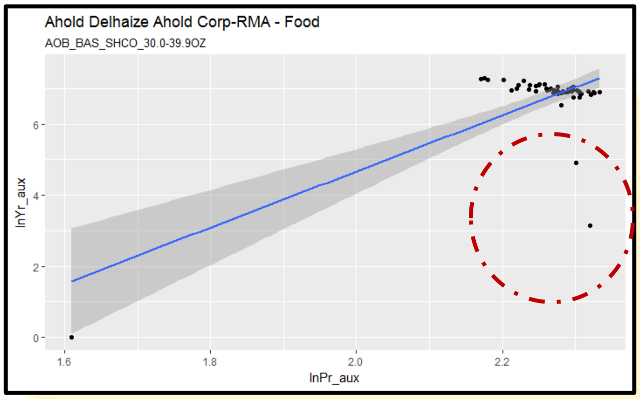
Figure 4: OLS (with and without structural break) estimation for lnQ. The breakpoint is observed at the 53th observation (x-axis) in the first week in January 2017 (1/8/2017), where lnQ equals to 7.57 (y-axis; quantity sold in this week is 1948 unites).

Figure 4 provides a better sense in terms of model fit using the structural breaks approach and without it. We see lnQ fitted with break (red line) follows the original lnQ (black line) closely, while the estimated lnQ based on regular OLS equation (blue line) missed the data behavior completely.

After dealing with break points potentially found in the data, there still remains to verify whether there are some outliers. I.e. a dataset can have both outlier and structure break problem or simply outlier issues.

To deal with outliers, we exclude the data that fall below the 0.05th percentile. This percentile can be adjusted in a case by case basis. However, in general this percentile appears to do a good work. Another way to solve this problem is to set up a

threshold point and adjust the lower and upper data values accordingly. A good visualization of this effect can be seen in Figure 5 with another product[12] that having outlier problem.
(a) OLS result before adjusting for outliers' effect, estimated elasticity equals to 7.908



(b) OLS result after adjusting outliers' effect, estimated elasticity equals to -2.10.

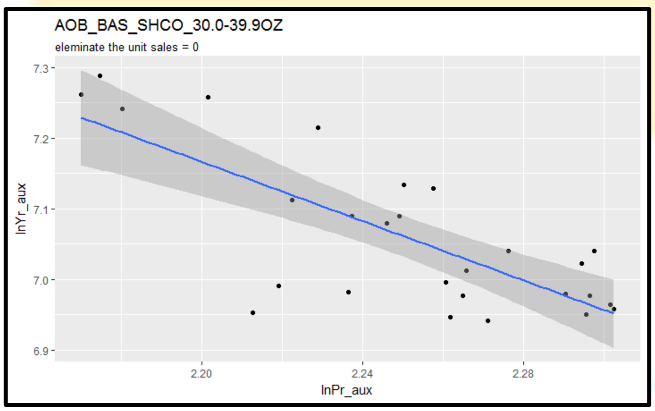


Figure 5: Comparisons of the OLS results for (a) whole dataset, (b) excluded outliers.

The main reason we get a positive elasticity result ($b = 7.908$) in Figure 5(a) is because a single observation located in the left bottom part of figure drives the real relationship between lnQ and lnP far away from the most likely best fit line.
In this case, beside the left bottom point, the other two black dots should also be counted as outliers. Since it diverges away from the cluster group[13].
After we exclude outliers, we can zoom in the cluster and uncover the real effect between these two variables. As shown in Figure 5 (b), with updated dataset, we get the price demand elasticity equals to -2.1, which is closer to the real value.

3.2 Quantile Regression (QR) with Outliers and/or Breakpoints

Koenker and Bassett (1978) come up with quantile regression (QR) approach to model conditional quantile based on a dependent variable. The objective function in the QR approach is to minimize a weighted sum of the absolute value of residuals. In mathematical form, the pth quantile estimators for (α_p, β_p) are chosen to:

$$min_{\alpha_p,\beta_p} \sum_{i=1}^{n} d_p(y_i, \hat{y}_i) = min_{\alpha_p,\beta_p} \sum_{i=1}^{n} \rho_p(y_i - \alpha_p - x_i'\beta_p)$$

$$= min_{\alpha_p,\beta_p} \left\{ p \sum_{i:y_i \geq \alpha_p + x_i'\beta_p}^{N} |y_i - \alpha_p - x_i'\beta_p| + (1-p) \sum_{i:y_i \leq \alpha_p + x_i'\beta_p}^{N} |y_i - \alpha_p - x_i'\beta_p| \right\} \#(7)$$

Applying the outlier and break points detection processes before performing the quantile regression analysis allows us to obtain better elasticity estimates. Using the same data[14] with breaks from the previous section, we can see in Figure 6 (a) that the positive lines in are biased output result from QR regression. Whereas after splitting the dataset into two parts based on the OLS break point estimates, the output significantly improved as seen in Figure 6 (b). For each dataset, we then split lnQ into different quantiles (0.2, 0.4, 0.6, 0.8). The blue lines present the estimated elasticity before break (dataset before 1/8/2017) in different lnQ quantile ranges and the red lines present the results in after break dataset. Coefficient results show in Table 1. At this time, elasticity in each quantile range has significantly improved.
(a) Quantile regression results before adjusting the structure break's effect.

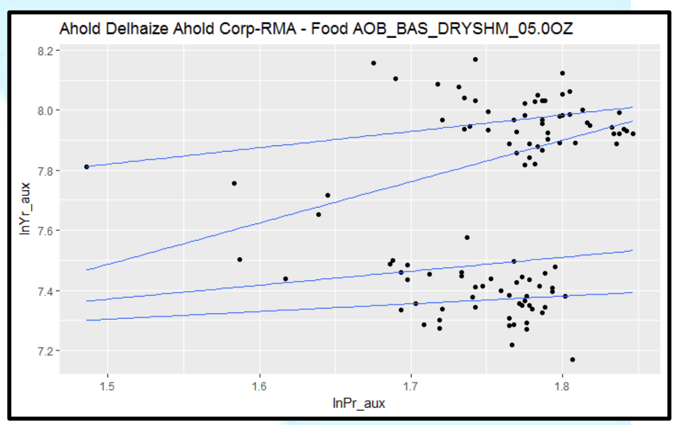
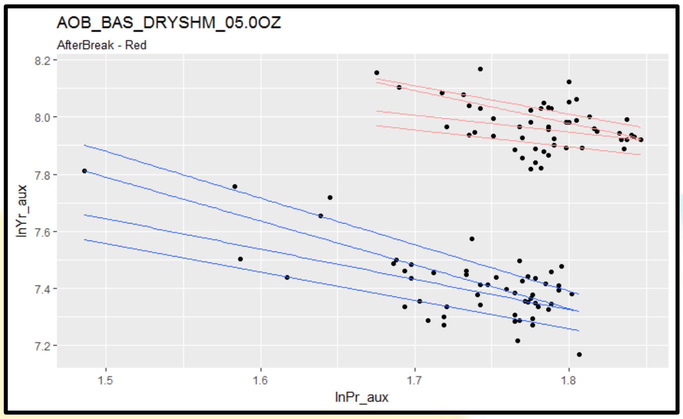


Figure 6: Comparisons of the quantile regression results for (a) before adjusting the structure break's effect, (b) after adjusting the structure break's effect.

12 For product AOB_BAS_SHCO_30.0-39.9OZ from retailer Ahold Corp.
13 For certain products in our datasets, mainly those new or discontinued ones, where there is a large number of weeks with no sales, we first eliminate these weeks and then apply the benchmark to find outliers.
14 For product AOB_BAS_DRYSHM_05.0OZ from retailer Ahold Corp.



(b) Quantile regression results after adjusting the structure break's effect. This table provides the coefficient estimates from the quantile regression approach. If we ignore structure break effect, the model returns all positive elasticities within each quantile. However, after considering structural breaks effect, QR reflects all negative results for data within each range which follows the law of demand. Note: the breakpoint is observed in the first week in January 2017 (the 53th observation, 1/8/2017).

Table 1: Quantile Regression Coefficients (Elasticity)

Coefficient	Tau 0.2	Tau 0.4	Tau 0.6	Tau 0.8	Coefficient	Tau 0.2	Tau 0.4	Tau 0.6	Tau 0.8
Whole Dataset	0.2555	0.4638	1.3753	0.5456	Before Break	-0.9971	-1.0608	-1.5319	-1.6291
					After Break	-0.5932	-0.5909	-1.1461	-0.9912

3.3 Quantile on Quantile Regression (QQR)

Traditional econometric models like OLS or threshold linear regression model can only consider certain relationships under average conditions and are not able to consider certain extreme events, neglecting in this way broader economic interactions. Alternatively, Quantile on quantile regression approach studies the joint co-movement between each different pair of (x, y). In our case, QQR approach is the only model that traces the elasticity changes given every different price and quantity combination. To be specific, instead of achieving one single elasticity result as from OLS, with QQR we can get an $n \times n$ elasticity matrix based on $n \times n$ different scales of price and quantity combinations. Given sufficiently large datasets, QQR model could uncover dynamic changes of elasticity and provide more valuable insights into market-promoting strategies for marketers and sale professionals.

Sim and Zhou (2015)[11] proposed quantile on quantile regression approach through the combination of quantile regression and local linear regression with first order Taylor expansion to express the dependency between different quantiles of dependent variable and different quantiles of explanatory variables. In this paper, we apply Sim and Zhou (2015)[11] QQR approach and update the QR equation accordingly.

In a regular QR equation, our regression function can be expressed as: $lnQ_t^\theta = \alpha_0^\theta + \beta^\theta(lnP_t) + \varepsilon_t^\theta$ #(8)

Where ε_t^θ is an error term in θ-quantile. We allow the relationship function $\beta^\theta(\cdot)$ to be unknown since we do not know about the way elasticity changes with different prices and quantity pairs. We then linearize the function $\beta^\theta(\cdot)$ by taking its first order Taylor expansion around τ-quantile of lnP to explore the link between the θ-quantile of lnQ and τ-quantile of lnP. With this we have: $\beta^\theta(lnP_t) \approx \beta^\theta(lnP^\tau) + \beta^{\theta'}(lnP^\tau)(lnP_t - lnP^\tau)$#(9)

Redefining $\beta^\theta(lnP^\tau)$ as $\beta_0(\theta,\tau)$ and $\beta^{\theta'}(lnP^\tau)$ as $\beta_1(\theta,\tau)$, equation (9) becomes:

$$\beta^\theta(lnP_t) \approx \beta_0(\theta,\tau) + \beta_1(\theta,\tau)(lnP_t - lnP^\tau)\#(10)$$

Substituting equation (10) into equation (8) to obtain the following: $lnQ_t^\theta = \alpha_0^\theta + \beta_0(\theta,\tau) + \beta_1(\theta,\tau)(lnP_t - lnP^\tau) + \varepsilon_t^\theta$#(11)

Since β_0 and β_1 are doubly indexed in (θ,τ) in equation (10), we now can analyze the whole joint co-movement distribution under each θ-quantile of lnQ given a different τ-quantile of lnP.

We employ a Gaussian kernel $K(\cdot)$ function to weight the observations in the neighborhood of lnP^τ, based on bandwidth h (we use 0.05 as recommended in Sim and Zhou (2005)). Therefore, the objective function to get quantile on quantile coefficient is:

$$(\hat{\alpha},\hat{\beta}) = argmin_{\alpha,\beta}\sum_{t=1}^{n}\left(lnQ_t^\theta - \left(\alpha_0^\theta + \beta_0(\theta,\tau) + \beta_1(\theta,\tau)(lnP_t - lnP^\tau)\right)\right)K\left(\frac{F_n(lnP_t) - \tau}{h}\right)\#(12)$$

Where $F_n(lnP_t) = \frac{1}{n}\sum_{k=1}^{n} I(lnP_k < lnP_\tau)$, $I(\cdot)$ is an indicator function and $K(z) = (2\pi)^{-0.5}e^{-z^2/2}$

In consistent with previous methodology, we detect outliers first and use the clean dataset to apply QQR approach. One benefit for using QQR is its inherent character for catching the structure break affect, considering it can fully capture the joint relation between two examined variables under each point of their respective distribution.

In this paper, limited observations per product (104 aggregated weekly observations), if there is no data falling in certain regions of the space of price and quantity combinations, we set the elasticity in that region to be zero to obtain robust results and easy to interpret graphs. We also exclude positive elasticity results from the output matrix since these are noises in QQR model.

3.4 Gravity Center Regression (GCR)

Gravity Center Regression is based on Nonlinear Nonparametric Statistics (NNS) and was developed by one of the authors. Using partial moments, we can partition the joint distribution of the data and create clusters that are hierarchical and partitional. By restricting the clusters to known elasticity properties (like negativity) in the upper left (Divergent Upper Partial Moment - DUPM) and lower right (Divergent Lower Partial Moment -DLPM) quadrants, we can estimate the true underlying elasticity signal in the aggregated noisy series.

For example, below is a visualization of the first order partitioning whereby most of the observations are in the DUPM and DLPM quadrant. This is consistent with a negative correlation coefficient as described in Viole and Nawrocki (2012)[14].

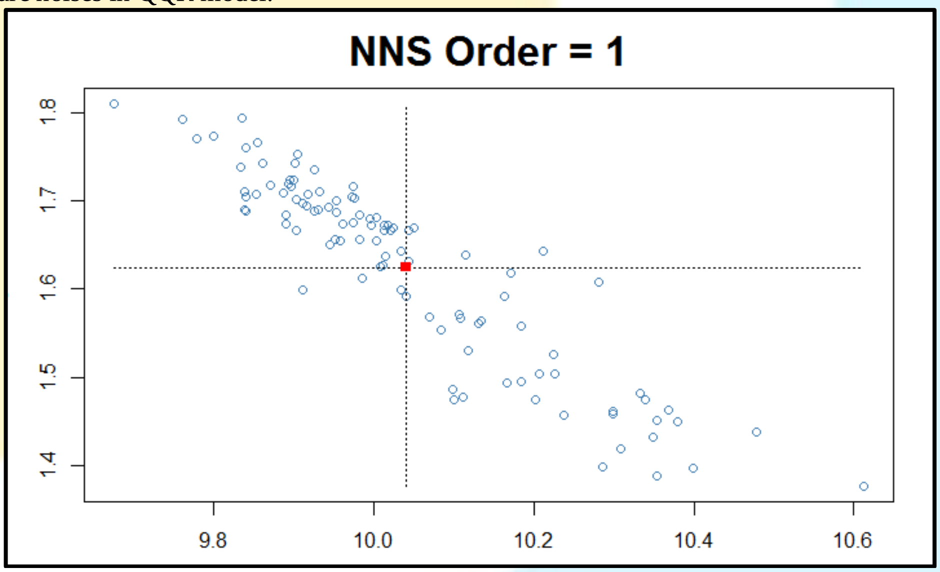


Figure 7: GCR approach for NNS partition of joint distribution.

The diverging lower partial moment ($DLPM$) and diverging upper partial moment ($DUPM$) matrices are defined by:

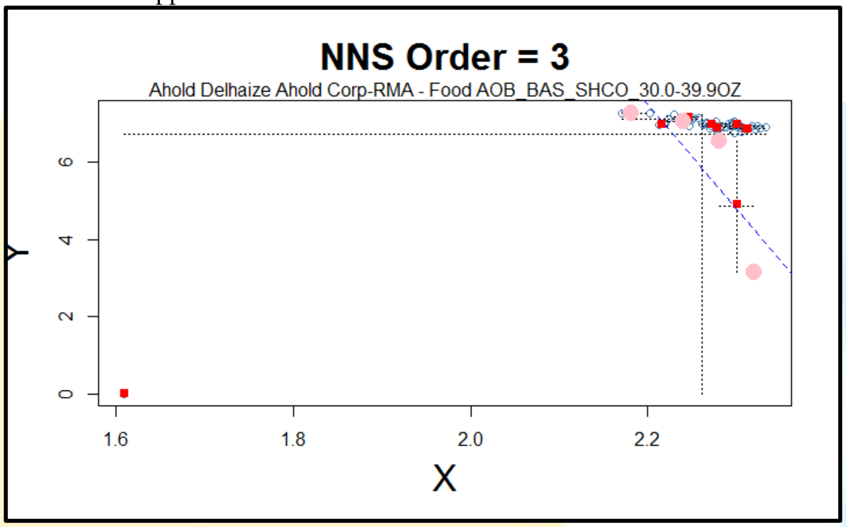
$$DLPM(n,h,x|y) = \frac{1}{T}\left[\sum_{t=1}^{T}(max\{x_t - h, 0\}^n \cdot max\{0, h - y_t\}^n)\right] \quad (13)$$

$$DUPM(n,h,x|y) = \frac{1}{T}\left[\sum_{t=1}^{T}(max\{0, h - x_t\}^n \cdot max\{y_t - h, 0\}^n)\right] \quad (14)$$

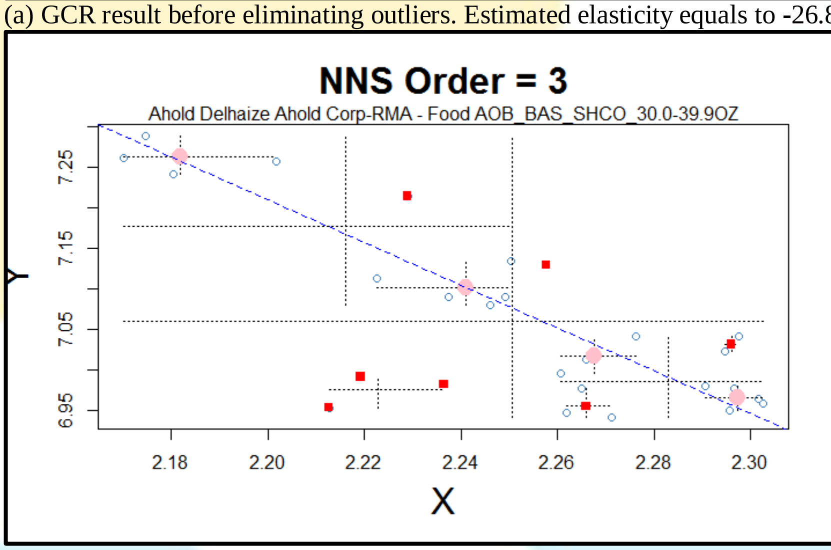
Equation (13) provides the divergent lower partial moment for variable Y given a positive target deviation for variable X from shared target h, with the degree (n). When $n = 0$, the partial moment matrices are a frequency statistic, while $n = 1$ is an area-based statistic. When the degree 1 divergent partial moment matrices are combined with the complement matrices of co-partial moments (CUPM and CLPM representing upper right and lower left quadrants respectively), we can recover the covariance between two variables.

The means of the resulting partial moment quadrants serve as the representative cluster for those member observations. These means (or other central tendency statistic such as medians or mode) serve as the basis of a nonlinear regression as described in Vinod and Viole (2017)[13]. However, in this application of elasticity, we are concerned with the overall coefficient, not the local coefficients Gravity Center Regression returns. Thus, we perform a simple linear regression on the partial moment clusters for our analysis.

Applying the outlier and break points detection processes ahead can further improve GCR output. Follow the same outlier example as we presented in Figure 5, Figure 8 (a) and (b) plot out the GCR results with and without outliers respectively. If outliers are included in the sample data, we achieve a negative slope ($b = -26.89$) with GCR positive result ($b = 7.908$). After removing several outliers, we achieve a negative slope ($b = -2.635$), which is close to the OLS approach without outliers approach compared with OLS ($b = -2.1$). Once more, note the importance of removing outliers before applying any model.



(a) GCR result before eliminating outliers. Estimated elasticity equals to -26.89.



(b) GCR result after eliminating the outliers. Estimated elasticity equals -2.635.

Figure 8: GCR results comparisons for (a) including outliers, (b) excluding outliers; Note: the large pink dots are the pairs of (lnP, lnQ) obtained from the Divergent-Partial Moments: DUPM and DLPM quadrants; red dots are the pairs obtained from the Co-Partial Moments: CUPM and CLPM quadrants. The blue dots are the (lnP, lnQ) pairs from our dataset after iteration and the blue line is the best fitting line (GCR regression line).

4. MODEL COMPARISON AND THE EMPIRICAL RESULTS

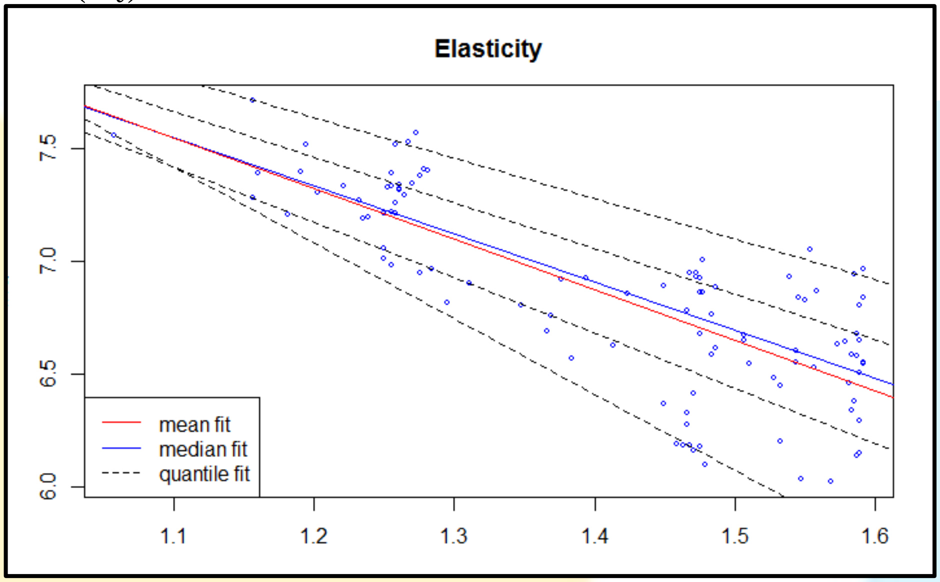
With 219,024 observations corresponding to 104 weeks (January 2016 to December 2017) for 340 Hair Care products sold in 11 retail stores, we apply OLS (Ordinary least squares), QR (quantile regression) and, thereafter QQR (Quantile on quantile regression) and GCR (Gravity Center Regression) approaches, conduct model comparison and result analysis, respectively. In this section we use a single product sold at Ahold Corp[15] as an example to present our findings and compare the results of the different models used.

4.1 Ordinary Least Squares (OLS) and Quantile Regression (QR) Comparison

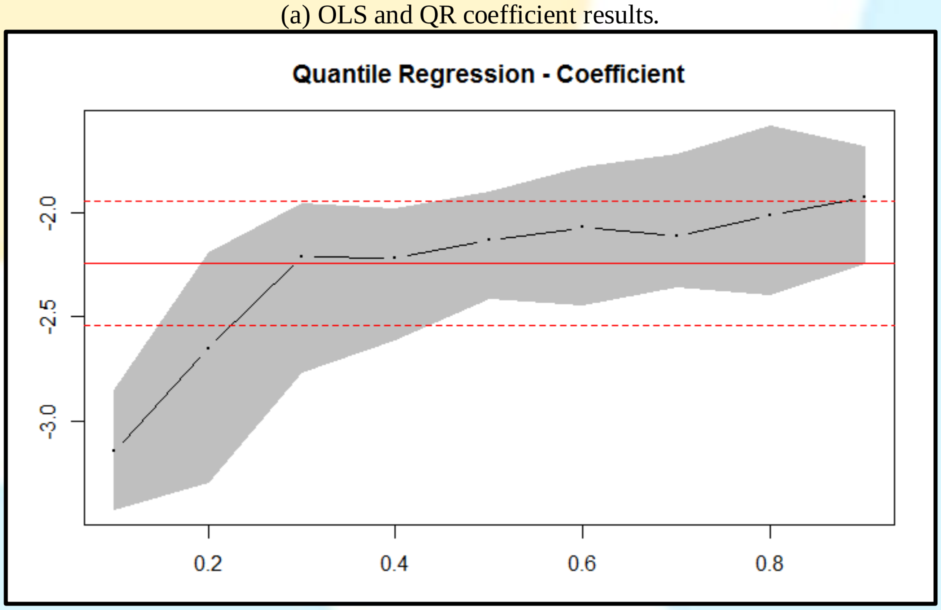
The OLS model presented in Equation (2) represents the change in the conditional mean of the dependent variable (lnQ) associated with a change in the explanatory variable (lnP). Even though this model provides good fit to well behaved data like the one presented in Figure 7, it provides an incomplete picture and might underestimate the effect of the covariates under extreme conditions (data with different EDRP regimes, outliers, among others).

[15] For product CRS_BAS_BDWS_13.5OZ from retailer Ahold Corp, aggregated from all of its locations.

Unlike the OLS model estimates, the QR improves the results by providing an $n \times 1$ output that captures the change of elasticity based on different quantiles of the dependent variable (lnQ). For a better sense of the difference between OLS and QR model, with lnP on the x-axis and lnQ on the y-axis, Figure 9 plots the results from these two models together.



(a) OLS and QR coefficient results.



(b) OLS and QR coefficient results under 95% confidence interval.

Figure 9: OLS and QR coefficient results. (a) Regression result from OLS conditional mean (red), QR conditional median (blue) and QR quantile fit (black); (b) coefficient from OLS model (red) and QR (black) under 95% confidence interval (OLS show in dashed-red lines; QR show in black color).

The grey lines in Figure 9 (a) represent the quantile fit based on lnQ in quantile 0.05, 0.25, 0.75, and 0.95 respectively; the red line shows the OLS conditional mean regression and the blue line is quantile regression based on conditional median.

Each black dot in Figure 9 (b) represents the estimated value of the elasticity coefficient for each of lnQ's percentile presented on x-axis. The grey shadow reflects the elasticity range within the percentile along with its 95% confidence level. The red line shows the OLS elasticity estimate with 95% confidence level (dashed-red lines).

Even though both OLS and QR provide negative elasticity results for this particular product, one can see that OLS conditional mean results significantly diverge from the ones estimated considering different percentile of unit sales (lnQ). The differences are larger under extreme conditions (lower percentile, Figure 9 (b)). Based on the QR output, the absolute value of price demand elasticity is higher when quantity is low (around -3) and decreases gradually when quantity increases (less elastic). Meaning, OLS could only reflect partial information to this case and

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ISSN: 2292-9355

special care needs to be taken since by far this model is the most widely used model in the industry.

4.2 Quantile Regression (QR) and Quantile on Quantile Regression (QQR) Comparison

Even though QR improves the elasticity estimates and provides richer results based on different quantiles of the dependent variable, the QR model only considers the dependent variable in different percentiles for all price levels, i.e. it does not consider the varying effect of the independent variable (prices). In other words, the QR model assumes that the elasticity is constant for all prices (lnP) given lnQ percentile (elasticity is a straight line under each percentile of lnQ). QR approach assumes that a one percent adjustment in prices is the same when current prices are low as well as when current prices are high, which is obviously not true. QQR model fills in this gap by considering the impact of different percentile price changes on the units sold according to their current price levels, i.e. a percentage change in prices when current prices are low has a different impact compared to an environment where the current prices are already high[16]. It is important to note that the results from QQR model is consistent with QR ones whenever the datasets are large.

We apply the QQR to the same product used before. The results are presented in Figure 10[17] with lnP on the x-axis and lnQ on the y-axis. According to this figure, the product is more elastic when price is high (bottom right part) and relatively less elastic when price is low (upper left part). Also note that the highest unit sales (in terms of lnQ in the y-axis) are those that correspond to lower prices, this is concordant with what is stablished by demand theory. This simply means that the product is more inelastic when the starting price (that could be current prices) are low, meanwhile when the price is already high, the product is more elastic and thus, changes in prices under this scenario have a more significant impact on unit sales.

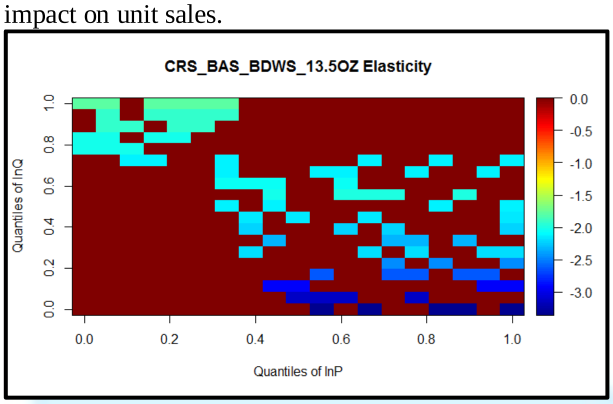

Figure 10: QQR coefficient results; quantiles of lnP is shown on the x-axis and quantiles of lnQ is shown on the y-axis. Dark red shows regions with no data; light blue represent elasticities at around -2 and, dark blue, elasticities in the -3 neighborhood.

From Figure 10, it is clear that the QQR results are consistent with QR findings. It is also interesting to note that the “trend” of the elasticity coefficients is increasing with price decreasing, satisfies the law of demand.

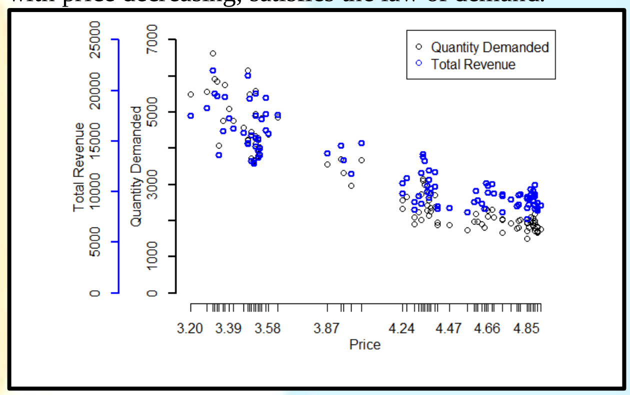

Figure 11: Relationship between price, quantity and total revenue; price in dollars is shown on the x-axis; pairs (price, quantity) showed as black dots; pairs (price, total revenue) showed as blue dots

Figure 11 further demonstrates that the total revenue increase with price decrease and for this product, the maximum total revenue happens when lnP is within its lowest quantile (when P is low) as in the upper left corner range. It is noteworthy that based on the dataset we have; this product is still within the elastic demand regime as seen in Figure 1. Continue to decrease price can push elasticity reach -1 and drive the total revenue up to its highest value.

Therefore, for this product, we could decrease the selling price which will help increase the quantity sold and from there the maximum total revenue point starts to build. Recall, that one maximizes total revenue when price elasticity equals -1. A note of care here, we are talking only about the total revenue function (defined as price times quantity sold) and not about profitability (defined as total revenues minus all costs).

In this paper, by using the QQR model for each product (only 104 observations per product), we have not been able to show all its benefits. However, given the authors' experience working with several other datasets with much richer dynamics (volatility) and most importantly, with more observations, we believe that the benefit that the QQR model is capable to provide significant information that other methods are not able to provide. The QQR results can help marketers to establish better strategies depending on current prices and observed dynamics between units sold and prices.

[16] We define low and high prices in reference to the product's own price dynamics. However, one can also think of low or high price relative to a substitute or complementary product.

[17] Note that what we get from the QQR model is almost identical to the results obtained with the QR more.

4.3 Ordinary Least Squares (OLS) and Gravity Center Regression (GCR) Comparison

Continuing the example presented in the previous section, we present the results obtained with the GCR model. We perform a fourth order partition of the joint distribution between lnP on the x-axis and lnQ on the y-axis, following Viole (2016)[15]. In Figure 12, the large pink dots are the pairs of (lnP, lnQ) obtained iteratively from the diverging lower partial moment (DLPM) and diverging upper partial moment (DUPM) quadrants to be consistent with the law of demand (the price – quantity relationship is negative, i.e. negative price elasticity). This methodology provides us with the benefit that in each iteration we only consider the data located in the relevant quadrants (DUPM and DLPM quadrants). In this way, unlike OLS linear regression model, the elasticity output from GCR model always follows the law of demand. In this case, elasticity based on the GCR model is -2.832 (compared to -2.243 from the OLS results).

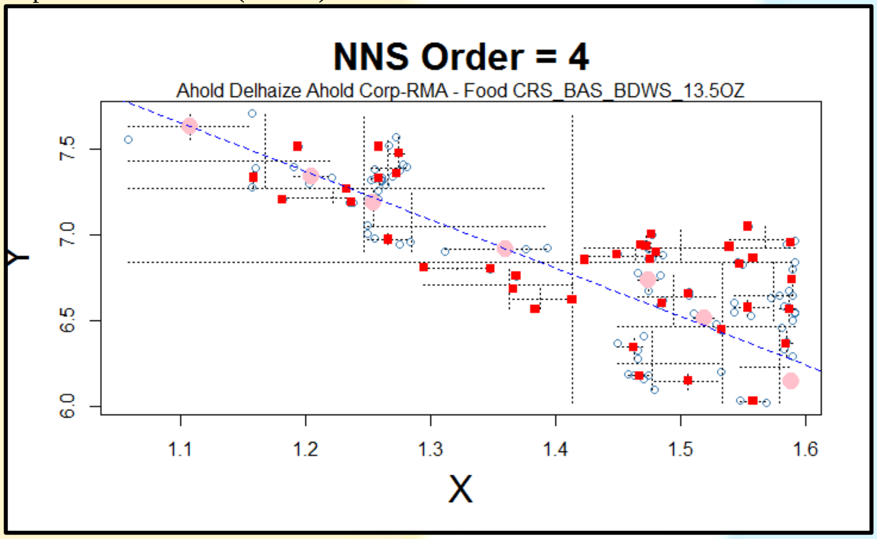

Figure 12: Visualization of DUPM and DLPM quadrants expanded. With lnP in the x-axis and lnQ in the y-axis, fourth order partition (NNS order equals to 4); the blue dots are the pairs of (lnP, lnQ); the large pink dots are the pairs obtained iteratively from the DUPM and DLPM quadrants; red dots are from CUPM and CLPM quadrants; regression line shows in blue; Estimated elasticity equals to -2.832 compared with OLS -2.243.

5. CONCLUSION

Price elasticity of demand plays a fundamental role in marketing strategies. A decrease in price will typically encourage consumer to buy more of this product and vice versa. Applying to market promotions, the marketers should understand whether the price of a product is in the elastic or inelastic regions and to understand how elasticity changes under different current price condition are important when developing an effective marketing campaign.

This paper shows how simple techniques can be used to eliminate measurement errors due to the presence of outliers or changes in EDRP's regimes. We conclude that applying outlier and breakpoint detection methods before applying any method significantly improves the results.

Our analysis is based on two years of weekly data (January 2016 to December 2017) for 340 Hair Care products sold in 11 retailers. We present four different econometric models: Ordinary Least Squares (OLS), Quantile Regression (QR), Quantile on Quantile Regression (QQR) and Gravity Center Regression (GCR), show their results and mentioned their main characteristics. The QQR model could catch the dynamic elasticity changes given each pair of price and quantity, and the GCR model is the only one providing consistent elasticity results that always follow the law of demand.

We left for future research the inclusion of other important variables such as competitors' prices. We are also committed to an ongoing process to improve the elasticity measurement process to yield more precise and accurate results that is necessary for enhancing our understanding of marketing strategies going forward.

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